Notes on Linear Algebra

# Cosine for of the Dot Product

We’ll demonstrate that the **cosine** form of the dot product can be derived from the **element-wise multiplication** form.

Where:



## The Law of Cosines

Based on the figure above we can write the following expressions:

From the definitions and , can be expressed in the right triangle with sides .

But:

This means the length of the difference vector can be written as:

Using the fundamental law of trigonometry::

|  |  |
| --- | --- |
|  | (1) |

Equation (1) represents ***the Law of Cosines***, which allows us to determine the length of the third side of a triangle when the lengths of the other two sides and the included angle are known.

## The element-wise multiplication – cosine form equality

The element-wise multiplication form of the Dot Product is:

The length of a vector can be expressed as the dot product with itself:

We know that the cosine form of the dot product is known as:

The law of cosines provides us with an equation of this exact form of the dot product, so that we only need to demonstrate that:

Rewriting the equation (1) gives us:

Conclusion: Using the law of cosines, we demonstrated that the cosine form of the dot product is equal to the element-wise multiplication form.

The dimensions do not matter because any 2 vectors will always represent 2 lines in higher dimensions.